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# Basics of Arabic Mathematical Language by Al-Khwarizmi

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***Abstract***
*Mathematics is one of the oldest sciences explored by nearly all human civilisations because of its direct connection with human life activities. Consequently, the remarkable intellectual advancement achieved in this field resulted from the cross-pollination of diverse human experiences, with the Arab contribution being particularly notable. Arab scholars have made significant efforts to develop many sciences, among which mathematics is especially prominent. The renowned scholar 'Al-Khwarizmi' became a leading figure in this domain, clearly impacting Arabic and global mathematics. His genius led him to establish new mathematical notation systems that continue to be utilised and engaged within modern mathematical research.*

***Keywords****: Algebra; Al-Khwarizmi; Mathematics; Mathematical notation; Mathematical terminology.*

## Les bases du langage mathématique arabe par Al-Khwarizmi

***Résumé***
*Les mathématiques sont l'une des plus anciennes sciences explorées par presque toutes les civilisations humaines en raison de leur lien direct avec les activités de la vie humaine. Par conséquent, les progrès intellectuels remarquables réalisés dans ce domaine résultent de la pollinisation croisée de diverses expériences humaines, la contribution arabe étant particulièrement remarquable. Les savants arabes ont déployé des efforts considérables pour développer de nombreuses sciences, parmi lesquelles les mathématiques occupent une place de*

*choix. Le célèbre savant « Al-Khwarizmi » est devenu une figure de proue dans ce domaine, influençant clairement les mathématiques arabes et mondiales. Son génie l'a conduit à établir de nouveaux systèmes de notation mathématique qui continuent d'être utilisés et engagés dans la recherche mathématique moderne.*

***Mots-clés** : Algèbre ; Al-Khwarizmi ; Mathématiques ; Notation mathématique ; Terminologie mathématique.*

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## Introduction

Mathematics stands out as one of the foremost sciences that humans have eagerly embraced, explored, delved into, and excelled across various branches, including arithmetic, algebra, geometry, and more. Humanity's engagement and fascination with mathematics stem from two pivotal reasons: their desire to uncover the mysteries of the universe and broaden the boundaries of fundamental knowledge. The second reason is the intrinsic connection of these precise knowledge systems with various practical human affairs, enabling individuals to benefit from and utilise mathematics to advance their material interests. Together, these factors have directly contributed to the effective growth of mathematical sciences, leading to their branching into numerous fields and specialisations. In this context, Arabs, like other nations, play a substantial role in developing Arabic mathematics. They benefited from the cultural exchange that helped shape its contours and, in turn, contributed to its advancement by establishing new concepts and correcting erroneous ones through numerous mathematicians, the most notable of whom was 'Al-Khwarizmi.' He left a significant mark on the pages of human mathematical thought with his innovations, which continue to demonstrate their greatness and unique role in constructing the foundation of mathematical science.

## 1. Definition of Mathematics

Specialised studies have yet to provide a stable, universally agreed-upon definition of the fields encompassed by 'mathematics,' which justifies the variations in its concept from one researcher to another, depending on the field in

which it is applied. This, naturally, is due to the multiplicity of its branches and domains.

Accordingly, mathematics is the abstract, logical study of systems and mathematical propositions. It addresses various issues, including measurement, geometry, arithmetic, dimensions, space, etc. In short, mathematics studies everything related to quantity, quality, and interconnection. Therefore, it requires specific intellectual skills, particularly analysis and reasoning, which must be precise, clear, and rational in posing and discussing issues. It is based on logic and proof and uses distinct and unique mathematical notation systems.

Some scholars define it on the basis of the methodology it uses to achieve its intended outcomes, describing it as an "abstract science that uses deduction to discover numerical and geometric relationships, among others. This science is characterised by its organisation, accuracy, and sequential presentation of information, which contributes to arriving at precise explanations of ideas and results. " (Rashed, M. 2009) Abstract methodology is the primary mechanism mathematicians employ in their studies and research. It distinguishes mathematics as a universal language on the basis of a set of shared symbols and rules, regardless of its origin and history of development. Conversely, others consider it a "high art due to its characteristic structure, sequence of ideas, harmony in construction, elegance in design, and precision in presenting mathematical models, whether scientific or illustrative of life situations." (**Ibid.,** p.15) The word 'Mathematics' has Greek origins. The concepts studied in this field have been expressed with various names throughout history and across different cultural milestones. In Latin and English, approximately 1700 BCE, the most common term for mathematical research was 'Astrology' or 'Astronomy,' but this

term eventually faded and evolved into the contemporary concept of mathematics. In discussing the historical origins of this word, Jacqueline Stedall stated:

> The Greek word 'mathemata' refers to what has been learned, sometimes in a general way, and at other times, it was more specifically associated with astronomy, arithmetic, or music. The modern term 'mathematic' and its equivalents in different European languages were derived from this Greek word. However, the meanings of words have undergone various changes over the centuries." (Stedall, J. 2016).

Thus, mathematics emerged through a series of human experiences that began at the dawn of history, ever after humanity's existence on Earth. It has undergone various stages, each contributing to its development and shaping it into its current form through the participation of diverse civilisations from East China and West China over extended historical periods.

## 2. Importance and origins of mathematics in human thought

Mathematics is one of the oldest sciences studied by scholars in various civilisations and has played a prominent role in human life since ancient times. This explains why it is one of the earliest sciences known to humankind, dating back to ancient times. Studies on the history and origins of this science have shown that the primary reasons for its emergence lie in human nature, the love for knowledge, and the passion for measuring natural phenomena. This curiosity, combined with familiarity with certain mathematical principles and concepts that emerged in the early stages of the field, enabled people to study the various natural phenomena around them and analyse their interrelationships.

This opened vast horizons for understanding the laws and standards governing the universe.

Therefore, mathematics is directly connected to practical, sensory reality and human daily practice, particularly in "Egyptian and Babylonian civilisations, which faced the economic and social pressures of calculating for trade activities and predicting astronomical events" (Al-Jabiri, M. A. 1998). Human activities and needs directly drove the emergence of new branches of mathematics in ancient civilisations. For example, "the floods of the Nile River pushed ancient Egyptians to invent geometric methods to measure field areas and organise agriculture and irrigation" (**Ibid.,** p.57). Furthermore, human interactions across different fields urgently require mathematical concepts and principles to manage and organise their affairs. An individual must interact with others, which requires understanding society and recognising its capabilities and circumstances. This is achieved through the significance of the numbers surrounding them. These numbers, and the associated social dealings such as buying, selling, giving, receiving, production, and consumption, are essentially built upon fundamental mathematical knowledge and skills. " (Rashed, M. 2009).

This implies that human cultural development leads to the advancement of mathematics and other sciences. The historical record proves that the emergence of mathematics did not occur all at once and was not solely the work of one individual or nation. "The Babylonians, Egyptians, Greeks, Indians, Arabs, and others... all made significant contributions to various fields of science, helping it grow and develop to its present state. " (Toukan, Q. H. n.d ), starting from the Sumerian civilisation and extending to the Arab–Islamic civilisation.

## 3. Stages in the Emergence of Mathematics in Arab-Islamic Civilisation

Mathematical sciences were prominent in Arab civilisation, where great efforts were made to develop mathematical concepts. This is evidenced by various pioneering theories introduced by early Arab scholars, reflecting their active engagement in this field. However, it is noteworthy that the term 'Mathematics' in its contemporary sense was not common in ancient Arab studies; the field was studied in various branches related to numbers, arithmetic, geometry, and others. These mathematical topics appeared in the Arab world under multiple names, commonly algebra, arithmetic, and trigonometry. Each mathematical concept differs in its study nature and applications, yet they share a unified focus. "The number of Arab mathematicians has been estimated at over five hundred, according to (Suter) in his book *The Arabs' Mathematicians, Astronomers, and Their Works*." (Mubarak, H., and S. Abu Khalil. 1991).

The Arabs were initially motivated by religious concerns to engage with certain mathematical aspects, which they employed to facilitate religious practices and ensure proper direction in worship. For example, they applied mathematical principles to regulate prayer times, determine the start of lunar months, and locate the qibla (direction of the prayer). They also used mathematical concepts to divide weights and spoils. Owing to their considerable focus on mathematics for organising and guiding religious life, these sciences experienced remarkable advancements. The development of Arabic sciences, particularly mathematics, can be divided into three primary stages:

***First: The Stage of Transmission and Translation***

The Arabs' initial endeavour in this fertile field of science was to turn to the intellectual contributions of previous civilisations that had engaged with mathematics. Their approach relied heavily on translation, as "Muslims translated Euclid's Elements—the cornerstone of the humanities—into Arabic, studied it thoroughly, expanded on its theories, devised intricate solutions, authored books following its structure, and introduced new issues unknown to ancient scholars." (Muhammad, M. A. A. M. 2018) They also applied mathematical concepts and skills to essential areas with close connections, such as architecture, astronomy, and geometry.

This translation stage spanned two centuries, from the 7th to the 9th century CE. They did not limit themselves to one nation or a specific line of thought in their persistent efforts. Instead, they draw upon a diverse range of scientific sources and methodologies. These included the general heritage of ancient Babylonian, Egyptian, Chinese, Indian, and Phoenician civilisations, which was transmitted through Islamic conquests and commercial exchanges. Additionally, the intellectual heritage of India and Persia contributed to numbering systems, whereas Greek heritage influenced the advancement of the geometric sciences. " (Zaghwan, B. H. 2014).

Consequently, the Arabs inherited an extensive intellectual legacy but did not merely adopt it. They made distinctive contributions in furthering and correcting some of its misguided directions.

***Second: The Stage of Purely Arabic Composition***

This stage differed from the previous one, as Muslims began developing distinct Arabic scientific thought across var-

ious fields of knowledge. They built upon the understanding of their predecessors, making valuable and enriching contributions to the field.

***Third: The Stage of Scientific and Intellectual Export***

In this stage, Arab sciences and knowledge spread from their birthplace in the Arab-Islamic world to Europe, experiencing the dark Middle Ages period. Europeans relied on Latin translations of various fundamental sciences in the Arab intellectual world. "Without the work of Arab scholars, European Renaissance scientists would have had to start from where the Arabs did, which would have delayed civilisation's progress by centuries." (Muntaser, A. H. 1970). Numerous Western scholars who have researched the history of sciences and the role of different human civilisations in their development have acknowledged this. Although some have denied the Arab role in Europe's scientific renaissance, the prevailing belief is that the Arabs introduced no new mathematical concepts and were merely involved in translation and transmission.

### 3.1. A Brief Overview of Al-Khwarizmi's Career and His Role in Advancing Arabic Sciences

Muhammad ibn Musa Al-Khwarizmi was born in Khwarazm at 780 CE but lived and settled in Iraq. Historical sources differ in their origins and identities. *The Mathematics Encyclopedia* and *Microsoft Encyclopedia* indicate that he was of Arab origin, whereas "other sources state that he was Persian or Turkish. " (Rashed, R. 1989).

Al-Khwarizmi was a polymath, excelling in multiple areas of knowledge, greatly aided by his position at the *House of Wisdom* in Baghdad, where he worked as a translator of for-

eign sciences and knowledge. The *House of Wisdom* was founded under Caliph Al-Ma'amoun's patronage, providing Al-Khwarizmi with an essential intellectual environment. Although he drew from many fields, he excelled in mathematics, particularly algebra and geometry. His reputation spread throughout the Arab and Western worlds through the mathematical books he authored, which were translated into multiple languages, immortalising his name in the annals of human thought. Historians noted that his works have 27 titles across various scientific disciplines.

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Arithmetic is a mathematical field involving numbers and arithmetic methods, including the four basic operations.

Some researchers refer to it as the 'science of numbers' or 'number theory,' a science addressing many social concerns, which explains its prevalence across most human civilisations. Arabs, too, had a keen interest in this field, as they "relied on the counting system for commercial transactions, inheritance distribution, land measurement, and in weight and volume calculations, as well as in the distribution of spoils..." (Toukan, Q. H. n.d ). Among the Arab contributions to 'number science' was Al-Khwarizmi's pioneering introduction of the (zero) symbol into the numerical system. Previous civilisations used numerical systems without employing this symbol or digit in calculations. One historian commented, "The Indians used the zero symbol to signify 'nothing.' Nevertheless, the new contribution by Al-Khwarizmi was that he gave zero value, solving the issue by assigning it as a multiple of ten, thus establishing place values of tens, hundreds, and thousands... The term zero, used by Europeans, derives from al-khwarizmi's terminology: Zero, Zero, Ziphirum, Ziffer, and capsicum, far. " (Osman, A. R. n.d. ).

All these arithmetic concepts are now included under what modern mathematics calls the 'decimal numbering system.'

One distinctive aspect of the scientific contributions made by Arab scholars was the systematic approach to presenting scientific material, whether old or new, opening vast opportunities for Western scholars to benefit from this refined, organised scientific repository. Al-Khwarizmi exemplified this methodology by writing a book on arithmetic that was the first of its kind regarding organisation, classification, and content. It was the first book to reach Europe, where it remained for a long time as a reference for scientists, merchants, and accountants, a source for their arithmetic stud-

ies. Readers may be surprised that arithmetic was known for several centuries by the name 'Algorismi,' after Al-Khwarizmi. (Toukan, Q. H. n.d.).

Thus, Western scholars became acquainted with the scientific contributions made by Al-Khwarizmi and other Arab scholars and benefitted from these intellectual treasures for centuries. Arithmetic reached Europe through *Al-Andalus*, where "this translation brought with it, in addition to the theory, the numbers known in the West as (dust numerals)." (Hamoud, K. 1990), whose trajectory we explore further in subsequent discussions.

**3.2. Trigonometry**

Trigonometry is a branch of mathematical geometry that studies angles and triangles. It examines the relationships between triangle sides and angles and their connections to other geometric shapes. Trigonometry is known among Arabs as 'the science of proportions' because it studies the relationships arising from ratios between triangle angles and sides.

The study of trigonometry has been conducted among previous civilisations, such as the Greeks, "who used these discoveries in their astronomical research, which is why trigonometry remained intertwined with astronomy for them. Indians also knew trigonometry, although their knowledge was more advanced than that of the Greeks" (**Ibid.,** p.91). Arabs later absorbed these insights after encountering the cultural heritage of earlier civilisations. In this context, Kamal Hamoud noted a significant Arab innovation in trigonometry: "The Arabs, after examining Greek and Indian knowledge, developed it further and added their innovations. Their first accomplishment was separating trigonometry from astronomy, then organising it and establish-

ing its principles." (Ibid**.,** p.91). Through these contributions, Arab scholars brought this science into existence after refining, correcting, and adding essential elements, leading many to consider it an Arab science (Toukan, n.d.). Al-Khwarizmi played an active role in systematically structuring its concepts, showcasing his exceptional genius.

### 3.3. Algebra

Ibn Khaldun described algebra as follows: "Algebra and Balancing is a branch of number science, a method by which unknown numbers are determined from known numbers when a relation is assumed between them." (Faroukh, O. 2002). Al-Khwarizmi focused his algebraic concepts on practical applications to facilitate life affairs such as sales, leases, currency exchange, and measurement.

In discussing Al-Khwarizmi's role in establishing mathematical principles in algebra, Qadri Toukan commented, "The Arabs invented an important, distinctive science that did not exist before them: algebra, invented by Muhammad ibn Musa Al-Khwarizmi. In Europe, this science is known as logarithm (logarithm), a term derived from al-khwarizmi's name—the founder of this science" (Toukan, Q. H. n.d. ). Numerous scholars have affirmed Al-Khwarizmi's precedence in establishing the foundations of this modern science. "Al-Khwarizmi introduced a science with complete autonomy, being the first to use the term *Al-jabr*, signifying a specific science with its rules, principles, and equations, and provided it with new technical terms to represent objects and operations. " (Hamoud, K. 1990).

In the same vein, he countered viewpoints that downplay Al-Khwarizmi's originality, casting him merely as an organiser of Greek and Latin ideas. He responded, “Concerning

the Western view of Al-Khwarizmi's originality, opinions are divided. However, objective research, which is free from religious or nationalistic bias, has confirmed that Al-Khwarizmi's algebra stands apart from the level found among the Indians and Greeks. Al-Khwarizmi was indeed the founder and pioneer of algebra (Ibid.**,** p.129).

One of Al-Khwarizmi's most famous works, *Al-Jabr wal-Muqabala*, became highly influential in the Arab world. Scholars consider it a comprehensive algebraic manual, making it an essential reference for anyone seeking to understand algebra, whether Arab or Western. "This book was translated into Latin in the 12th century by several translators, including Gerard Cremoni, and later into Italian, German, and English, eventually becoming a core mathematics reference in most Western universities in the 15th and 16th centuries" (Ibid., p.138). In the Arab world, numerous scholars have studied and commented on its content, benefiting from its intellectual wealth, a legacy that remains celebrated in human civilisation today.

## 4. Mathematical Language in al-khwarizmi's Book *Al-Jabr wal-Muqabala*

The 'language of mathematics' is defined as the notational system used by mathematicians to convey their ideas and concepts to others. This linguistic style has specific (Ibid., p.129) characteristics, as it does not rely on the communication systems of natural human languages. However, it incorporates certain linguistic elements, precisely the mathematical terms often derived from general language. In addition, it employs a precise symbolic system that grants it expressive specificity for the mathematical formulations on which it is based. This language also relies on counting and

numbering systems, a necessity given the quantitative and calculative concepts it addresses.

This study aims to highlight the features of mathematical language within Arab thought by examining its key attributes and foundations through the work of Al-Khwarizmi—a pivotal figure in Arabic mathematics regarded by researchers as the progenitor of a new branch of mathematics. Al-Khwarizmi played a foundational role in uncovering the principles of this science, and we examine three core aspects of the language of algebra and mathematics, as presented in his book, *Al-Jabr wal-Muqabala*, which represents the initial model of Arabic algebraic principles.

***First: Technical Terms (Mathematical Terminology)***

Like other sciences, mathematics is distinguished by a specialised terminological system that sets it apart from different knowledge domains. Al-Khwarizmi was among the scholars who ventured into terminology development: "he was the first to coin the term 'al-jabr,' denoting a defined science with its rules, principles, and equations, and he enriched this science with new technical terms to represent objects and processes. " (Ibid., p.129).

In its early stages, Al-Khwarizmi's algebra was devoid of symbolic notation. Historical studies reveal that this form of notation was seen in Arabic mathematical writings in a later period, with early scholars relying on linguistic terms from the general lexicon, which were later endowed with narrower mathematical meanings.

In *Al-Jabr wal-Muqabala*, Al-Khwarizmi described the fundamental principles of algebra in a unique linguistic style that reflects the originality of his algebraic research. Despite his extensive translation work from these civilisations, he

used Arabic, which was unaffected by translation, and was free from Greek or Persian influences.

Examining Al-Khwarizmi's mathematical terms reveals that they originally come from a general vocabulary but semantically align with the mathematical concepts they represent. The historian of science Rushdi Rashed described Alkhwarizmi's approach to terminology: Al-Khwarizmi used existing words, retaining some meanings while assigning technical meanings to others, such as expressions for arithmetic operations or whole and fractional numbers. These terms were the same as those used by Al-Khwarizmi's predecessors and had similar meanings. However, some terms, such as 'shay شيء ' (thing), belong to everyday language but gain technical meaning." (Rashed, M. 2009). Rashed noted that Al-Khwarizmi drew most of his terminology from the linguistic and literary resources of his time or earlier periods, describing the language in *Al-Jabr wal-Muqabala*: "The mathematical language in this book's section, as in other sections, came from the pre-Al-Khwarizmi lexicon and literary works of his predecessors. (Ibid., p.60).

Thus, Al-Khwarizmi's algebraic terminology formed a precise terminological system that constitutes the core around which the mathematical concepts in *Al-Jabr wal-Muqabala* revolve. Kamal Mahmoud explains, "Al-Khwarizmi discusses numbers and their types, or primary terms, which we call terms in modern algebra. He defines them in three types: roots, squares, and single numbers unrelated to roots or squares." (Hamoud, K. 1990). Some contemporary researchers have equated al-khwarizmi's terms with modern terms, adding symbolic notation:

- **Root**: What we now call the unknown (x), termed 'shay شيء' in algebra, is the unknown term in the equation.

- **Square**: The root multiplied by itself, or the square of the root = \(x^2 \).

- **Single**: A number without a root or unknown.

-**Part of the thing**: The inverse of the root, or \(\frac{1}{x} \) (Ibid., p.120).

These fundamental expressions form the basis of Al-Khwarizmi's equations, which he formulated without symbols. For example, he writes, "One hundred and a square minus twenty roots plus fifty and ten roots minus two squares is one hundred and fifty minus a square and minus ten roots. " (Al-Khwarizmi, M. ibn M. 1937) Some terms, such as 'multiplication' and 'addition', remain in the Algerian educational system, whereas others, such as 'thing' and 'square', have fallen out of use. Still others have evolved in meaning but continue in educational contexts, especially terms such as 'roots' and 'single.'

***Second: Algebraic Notation***

Notation systems are critical components of mathematical expression and exceed the communicative goals of standard linguistic systems. Symbols represent a high level of intellectual abstraction, allowing for the conversion of linguistic representations of logical propositions into purely mathematical forms that are easy to use. Additionally, their universal adaptability facilitates cross-language learning and is noted for precision and consistency. (Ali, M. A. M. n.d. ).

Although mathematical notation development lagged behind the evolution of mathematical inquiry, Arabs were ahead of the West in using notation to express their mathematical concepts. Evidence from scientific Arabic writings shows that Arab mathematicians utilised this method in their works, such as Abu Al-Hasan Al-Qalsawi, who used

the first letter of 'jidr جذر ' (root) as a symbol (√), ' شيء' (thing) as 'x' for the unknown, 'maalمال ' (money) for \(x^2 \), 'ka'b كعب' (cube) for \(x^3 \), and 'laam ل ' for equality (=) (Atiyya, A. A. H. 1991).

In recent years, these symbolic models have been removed from Algerian educational curricula and replaced by foreign symbols resembling graphical forms or letters from French or English alphabets. In earlier years, the Arabic notation system was used throughout all levels of education, except at the university level, where foreign languages dominate the teaching of this scientific subject, as seen in the following examples.

**First Example:** Extracted from the *Mathematics Book* for the Fourth-Grade Middle School Level

In this example, concepts are presented in Arabic, which serves as the language of instruction for specialised concepts in these books. This is evident in the following excerpt. (Sharabta, B. 2019)

Given a triangle ABC, AD is the median related to the side BC.

Let M be a point on [AD], which is distinct from A and D. The line that includes M and is parallel to (AC) intersects [BC] at point E. Similarly, the line that includes M and is parallel to (AB) intersects [BC] at point F.

1. Draw an appropriate diagram.

2. We prove that DE/DC = DF/DB, deducing that D is the midpoint of [EF].

In earlier stages, the Algerian educational system relied entirely on the Arabic language to present and explain mathematical concepts included in the scientific curricula across various educational levels. The example presented here tests this practice.

***Definitions and Symbols***

Consider the set 'مج' (M) representing a specified classroom. To express that a student 'ع' (A) belongs to this class or that it is an element of this class, we write:

ع ∈ مج

where the symbol '∈' represents membership.

If the elements of 'مج' (M) are defined as {أ، ب، ج، د} (A, B, C, D), then we can write:

مج = {أ، ب، ج، د}

If the set 'مج' (M) does not include the element 'أ' (A), we write:

أ ∉ مج

indicating that 'أ' (A) is not a set member.

If the number of elements in the set 'مج' (M) is undefined and these elements satisfy certain conditions, we can also use braces. For example, to represent the set of even integers more significant than the number 5 integers, we write the following:

مج = {س | س is an even integer, س > 5}

***Second Example***

Upon reviewing Al-Khwarizmi's mathematical books, notably his work *Al-Jabr wal-Muqabala*, we observed a complete absence of a notation system. Instead, he employed standard language to convey his concepts while relying on specialised terms, some examples of which we previously discussed.

***Third: Numeration System (Arabic Numerals)***

A mathematical numeral is 'the symbol used to represent a basic number, which includes the first nine numbers and zero.'[1]. Numerals hold great significance in mathematics and daily social transactions; thus, various human civilisations

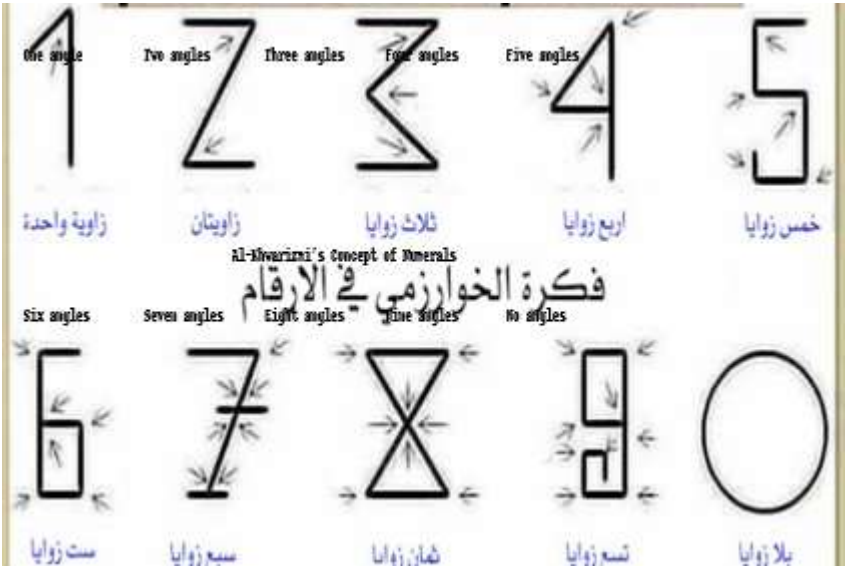
recognised and developed their unique numeral systems throughout history.

Arabs, like other civilisations, used this system in their transactions and their mathematical writings. "They expressed numbers in words written entirely, as seen on the 'Namara Stone' found among the ruins of Namara (in Hauran), dated to 328 AH. This is further confirmed by the 'Abraha Al-ashraf' text engraved on the famous Marib Dam (Ibid., p.13). Moreover, the Arabs applied a system known as 'Hesab Al-Jummal' (Abjad numerals), where numbers are represented by letters of the alphabet (أبجد هوز خطي كلمن). They utilised various counting systems, including 'finger counting' or 'hand calculations,' where different hand gestures represent numerous numbers. This system is an authentic Arab creation not borrowed from other nations (Ibid., p.19).

Moreover, owing to Al-Khwarizmi's efforts, Arabs were also introduced to Indian numerals. Al-Khwarizmi fundamentally modified the Indian numeration system, which initially lacked the number (0), as noted earlier. Through translating Indian mathematical texts into Arabic, he refined the system, which was then known as the 'Dust Numerals.' The Arabs named it so because they observed the Indians using a wooden board covered with fine dust for calculations (Ibid., p.30). Al-Khwarizmi did not merely adopt the original Indian numerals; he enhanced and refined them, creating what became known as 'Arabic--Indian numerals.' "One of the greatest contributions Arabs made to the world was introducing the Indian counting system and refining the Indian numerals, now commonly known as Arabic numerals in the West, as they were introduced to Europeans via Al-Andalus. " (Toukan, Q. H. n.d. ).

Al-Khwarizmi based the design of Arabic–Indian (dust) numerals on the number of correct and acute angles each numeral represents. For example, the number (1) has one angle, (2) has two angles, and so forth, as illustrated below:

**Figure 1:** ***Al-Khwarizmi's angular numeral system: an early conceptualisation of Arabic numerals***




Al-Khwarizmi aimed to provide specific symbols that people could use as alternatives to words and letters in various aspects of life through his numeral design modifications. The influence of Al-Khwarizmi's Arabic numerals reached Europe, where they were known as "Arabic numerals." In recognition of the Arab contribution in this area, German orientalist Sigrid Hunke states, "The Arabs' numbers, their instruments, which they perfected almost completely, their calculations, algebra, and science... all are Arab gifts to the

West that elevated Europe to a position enabling it, through its inventions and discoveries, to lead the world." (Hunke, S. 1992). This is evidence that Europe was among the greatest beneficiaries of Arabic thought in numeration. Arabic numerals remain the standard numeric system used in Western countries, while their primary use in the Arab world is now concentrated in the Maghreb region. In contrast, Arabs in the Levant and Eastern Arab countries have adopted mainly what are known as "Eastern Arabic numerals," which are of Indian origin.

## Conclusion

From the preceding discussion, it is evident that Al-Khwarizmi is among the greatest and most esteemed mathematicians in the history of human thought. He played a significant role in establishing the principles of algebra, subjecting it to an unprecedented scientific methodology. His work was structured in an organised, classified, and refined system, and he also made notable contributions to the numeration system, which became the foundational pillar on which various mathematical concepts are built. Modern mathematics, both in the Arab and Western worlds, still employs this system today.

In summary, Al-Khwarizmi is a series of Arab scholars who left remarkable contributions across various fields of knowledge, opening vast avenues for human civilisation to advance and progress. Although many voices deny this truth, claiming that Arabs merely transmitted the intellectual heritage of earlier civilisations, they overlook that civilisation is a continuous chain, with each link building on its predecessors. The Arab contribution, particularly embodied

in the genius of Al-Khwarizmi and his followers, preserved and expanded upon the Greek, Indian, and other heritages, ultimately transmitting this invaluable knowledge to the West and many of their innovations.

## Bibliographie

Ali, M. A. M. n.d. *Philosophy of Science* (Vols. 1–2). Beirut: Dar Al-Nahda Al-Arabiya.

Al-Jabiri, M. A. 1998. *Introduction to the Philosophy of Modern Rational Science and the Evolution of Scientific Thought* (Vol. 4). Beirut: Center for Arab Unity Studies.

Al-Khwarizmi, M. ibn M. 1937. *Al-Jabr wa Al-Muqabala*. Paul Barbier Press, Faculty of Sciences, Egyptian University No. 02.

Atiyya, A. A. H. 1991. *Studies in the History of Science among the Arabs* (Vol. 1). Egypt: Dar Al-Thaqafa.

Faroukh, O. 2002. *History of Science among the Arabs*. No Edition.

Hamoud, K. 1990. *History of Science among the Arabs* (Vol. No Edition). Lebanon: Dar Al-Fikr Al-Lubnani.

Hunke, S. 1992. *Arab Sun Shines on the West: The Influence of Arab Civilisation on Europe*. Translated by Farouk Baydoun and Kamal Dsouki. Beirut: Dar Al-Jeel.

Mubarak, H., and S. Abu Khalil. 1991. *The Role of Arab-Islamic Civilisation in the European Renaissance* (Vol. 1). Damascus: Dar Al-Fikr.

Muhammad, M. A. A. M. 2018. "The Role of Muslim Scholars in the European Renaissance." *Journal of the Faculty of Islamic and Arabic Studies for Women*, Issue 2, Vol. 3.

Muntaser, A. H. 1970. *The Impact of Arabs and Islam on the European Renaissance*. Egyptian General Authority for Authorship and Publishing.

Osman, A. R. n.d. "Contributions of Al-Khwarizmi in Algebra." First World Conference on the History of Applied and Medical Sciences among the Arabs. Riyadh: Imam Muhammad ibn Saud University.

Rashed, M. 2009. *Mathematics Curricula and Teaching Methods for Primary Sections* (Vol. 1). Jordan: Dar Al-Handareya.

Rashed, R. 1989. *The History of Arabic Mathematics: Between Algebra and Arithmetic*. Center for Arab Unity Studies.

Sharabta, B. 2019. *Mathematics (Fourth Year Middle School)*. Algeria: National Office of School Publications

Stedall, J. 2016. *A History of Mathematics* (Vol. 1). Translated by M. A. Sadaoui. Hindawi Foundation for Education and Culture.

Toukan, Q. H. n.d. *The Arab Scientific Heritage* (Vol. 3). Cairo: Shorouk Press.

Zaghwan, B. H. 2014. "Contributions of Arab and Muslim Scholars in Basic Sciences and Their Impact on the Building of Scientific Civilisation." Paper presented at the International Conference on the Arabic Language (Investment in the Arabic Language and Its National, Arab, and International Future), Dubai, United Arab Emirates.